# MESON PHOTOPRODUCTION AT THRESHOLD IN THREE FLAVOR SOLITON MODELS

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**Abstract**

In Skyrme-type models, the leading term of the low-energy photoproduction amplitude is identical to the standard expresssion and independent of the number of flavors considered, but subleading terms are not.

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Recently it has been shown, that a wide class of topological soliton models within $SU(2)$ lead to the well-known[1] low-energy theorem of pion photoproduction derivable up to $\mathcal{O}(N_C^{-1})$ and $\mathcal{O}(m_\pi)$, ref.[2]. Generalization of such models to more flavors is quite straight-forward in Skyrme-type models and much effort has been spent, for instance, on the properties of baryons rotating in $SU(3)$, see e.g. ref.[3]. In this little note I wonder, whether the low-energy theorem is altered, when further flavors are added to the $SU(2)$ soliton. Also, it might be possible to make additional statements on meson production channels beyond the four traditional pion-nucleon channels, when the photon either strikes a non-strange baryon or produces strangeness in the meson-baryon exit channel.

In soliton models the meson-baryon-photon interactions follow exclusively from the vector and axial vector currents inherent in the purely mesonic Lagrange density $\mathcal{L}(U, \partial_\mu U)$. These currents are defined as

$$V_a^\mu(U) = -i\,\mathrm{tr}\,\frac{\delta\mathcal{L}}{\delta\partial_\mu U}[\frac{\lambda_a}{2}, U]\,, \qquad A_a^\mu(U) = -i\,\mathrm{tr}\,\frac{\delta\mathcal{L}}{\delta\partial_\mu U}\{\frac{\lambda_a}{2}, U\}\,. \tag{1}$$

For definiteness I also list the currents from the chiral anomaly

$$\begin{aligned} V_a^\mu(U) &= \frac{N_C}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\,\frac{\lambda_a}{2}\Big\{-U^\dagger\partial_\nu U U^\dagger\partial_\rho U U^\dagger\partial_\sigma U + U\partial_\nu U^\dagger U\partial_\rho U^\dagger U\partial_\sigma U^\dagger\Big\} \\ A_a^\mu(U) &= \frac{N_C}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\,\frac{\lambda_a}{2}\Big\{\;\; U^\dagger\partial_\nu U U^\dagger\partial_\rho U U^\dagger\partial_\sigma U + U\partial_\nu U^\dagger U\partial_\rho U^\dagger U\partial_\sigma U^\dagger\Big\} \end{aligned} \tag{2}$$

If the parameters $\phi$ of a chiral rotation

$$U = e^{\frac{i}{2}\phi\cdot\lambda}\,U_0\,e^{\frac{i}{2}\phi\cdot\lambda} \tag{3}$$

are introduced, the following identity can be shown to hold for arbitrary $U_0$

$$V_a^\mu(U) = V_a^\mu(U_0) - f_{abc}\,\phi_b\,A_c^\mu(U_0) + \mathcal{O}(\partial\phi, \phi^2) \tag{4}$$

for all parts of the current originating from chirally symmetric terms of the action.

In Skyrme-type models, the nucleon is based on topological field configurations of the hedgehog soliton embedded in the isospin subgroup $SU(2) \subset SU(3)$

$$U_H(\boldsymbol{r}) = e^{i\tau\cdot\hat{r}\chi(r)}\,. \tag{5}$$

Kinematical and spin degrees of freedom are introduced by collective coordinates. For small velocities such coordinates can originate from a Galilean transformation of the center of mass $\mathbf{r} \to \mathbf{r} + \mathbf{X}$ or from adiabatically slow flavor rotations $A\,U_H\,A^\dagger$.

In the low-energy region of interest here, meson-baryon S-wave configurations can elegantly be written as

$$U = e^{\frac{i}{2}\lambda\cdot\phi}\,A\;U_H\Big(x_i + \frac{3g_A}{2M}\phi_a D_{ai}(A)\Big)\;A^\dagger\,e^{\frac{i}{2}\lambda\cdot\phi}\,. \tag{6}$$

where at threshold a meson is introduced as the term linear in the angles $\phi$ of a chiral rotation. Actually, it represents a linear combination of a chiral rotation of the hedgehog and a translation, $X_i = \frac{3g_A}{2M}\phi_a D_{ai}(A)$. (As usual we use $D_{ab}(A) = \frac{1}{2}\,\mathrm{tr}\,\lambda_a A^\dagger \lambda_b A$. The index $i \in \{1,2,3\}$ stands for the three spatial directions). In case of chiral symmetry, i.e. when chiral symmetry breaking mass terms are absent, both modes, translation and chiral rotation are zero frequency solutions to the adiabatic equations of motion for small amplitude fluctuations. The special linear combination given here is determined by the fact, that the S-wave scattering solution is orthogonal on the localized, purely translational zero mode. The overlap integrals of infinitesimal fluctuations between chiral rotations and translations involve the mass $M = -L[U_H]$ of the soliton and its axial coupling $g_A$,

$$\begin{aligned} \int A_a^j \left(AU_H A^\dagger\right) d^3r &= D_{ab}(A) \int A_b^j\,(U_H)\, d^3r \\ &= \left\{ \begin{array}{ll} -\frac{3}{2} D_{aj}(A)\, g_A^{SU(2)} & \mathrm{for} A \in SU(2) \\ -\frac{30}{14} D_{aj}(A)\, g_A^{SU(3)} & \mathrm{for} A \in SU(3) \end{array} \right. . \end{aligned} \tag{7}$$

The proton-neutron matrix elements of the axial current define the isovector axial charges and from this definition we deduce for $SU(3)$-symmetric nucleon states

$$g_A^{SU(3)} = \frac{7}{10} g_A^{SU(2)}, \qquad g_A^{SU(2)} = g_A\,. \tag{8}$$

The low-energy production amplitudes now follow directly from the lagrangian linear in the photon $a_\mu$ and the meson field $\phi$.

$$L_{\gamma\phi} = L_{\gamma\phi}^V + L_{\gamma\phi}^{ano} = -\mid e \mid \int a_\mu \left[V_e^\mu(U)\right]_{lin} d^3r \tag{9}$$

As usual, we abbreviate $(e) = (3) + \frac{1}{\sqrt{3}}(8)$ for electromagnetic charge indices. At low energies, in Coulomb gauge, only the constant polarization vector $\mathbf{a} = -\frac{1}{4\pi}\boldsymbol{\epsilon}$ is important. In contrast to the $SU(2)$-case, the contribution of the anomaly in $SU(3)$ leads to an additional vector current associated with the eighth generator, whereas the $SU(2)$ expression involves the topological winding number current. The main production amplitude at threshold, as in $SU(2)$, comes from the isovector charge of the mesons, however, and may easily be obtained via eqs.(4,7) as

$$L_{\gamma\phi}^V = -\frac{\mid e \mid}{8\pi} 3 g_A\, f_{ebc}\, \phi_b\, D_{ci}(A)\, \epsilon_i\,. \tag{10}$$

The only difference relative to the two flavor case is the appearence of $f_{ebc}$ instead of $\epsilon_{3bc}$ and the $SU(3)$-D-function, the nucleon matrix elements of which are changed from $-\frac{1}{3}\tau_c\sigma_i \overset{SU(3)}{\rightarrow} -\frac{7}{30}\tau_c\sigma_i$ for $SU(3)$ symmetry; since the differing factor of $\frac{7}{10}$ is absorbed into the definition of the nucleon axial charge in the three flavor case, eq.(8), this isovector amplitude is identical to its two-flavor partner when pions are produced on non-strange baryons. Checking out other mesons and baryons we

find that only charged mesons can be produced here, $\pi_0, \eta, K_0, \bar{K}_0$ do not couple in this lowest order Kroll-Ruderman amplitude.

Corrections linear in the meson mass arise for winding number $B = 1$ from the anomalous current. In contrast to the two flavor case there are two different contributions, the first one being an immediate generalization of the two-flavor isoscalar current

$$L_{\gamma\phi}^{(ano,1)} = \frac{\mid e \mid}{8\pi} \frac{3g_A}{2M} \dot{\phi}_b D_{bi}(A)\epsilon_i \, B \, \frac{N_C}{\sqrt{3}} D_{e8}(A) \tag{11}$$

as may be checked by taking the $SU(2)$-limit $D_{38}(A) \to 0, D_{88}(A) \to 1$. In $SU(3)$ this term allows for the production of neutral mesons just as the other, novel term does (we abbreviate $k, k' \in \{4, 5, 6, 7\}$):

$$L_{\gamma\phi}^{(ano,2)} = \frac{\mid e \mid}{8\pi} \frac{N_C}{9} d_{ikk'} D_{ek}(A) \dot{\phi}_b D_{bk'}(A) \, \epsilon_i \int r \, s \, B^0(U_H) d^3 r. \tag{12}$$

The novel term is no longer expressible by the charges $g_A, B$ of the baryon but involves an integral over the winding number density $B^0$ of the soliton together with a trigonometric function of the chiral angle ($r\,s = r\,\sin\chi(r)$).

The three different production amplitudes, eqs.(10,11,12), constitute all possible terms in $SU(3)$-symmetric soliton models up to linear order in the meson masses and zeroth order in the rotational velocities of the soliton. How useful is such a classification? In soliton models $SU(3)$ symmetry is broken by two different kinds of terms:

(i) meson mass terms which split the $K$ and $\eta$ masses from the pion. Since they involve no gradients of the chiral fields, they do not lead to additional photocouplings, their sole effect being a distortion of the hedgehog fields and the chiral rotation around them. Their contributions are always quadratic in the meson masses with exception of the time dependence of the produced (fluctuating) mesons, which at threshold also has a linear term because of the quantization of the fluctuations. We will examine the implicit effects of this flavor and chiral symmetry breaking on the threshold amplitudes.

(ii) Kinetic symmetry breakers, which lead to different weak decays of the mesons, is the other kind of terms. They will lead to further photocouplings. Generally, they are numerically less important and will be omitted in the present considerations.

It is clear that the $K$ and $\eta$ masses are not small numerically, thus the amplitudes in eqs.(10,11,12) are not trustable numerically, but they represent a systematic expansion of the threshold amplitudes, which certainly is interesting in itself, and it covers the whole octet of baryons in the incoming channel together with the octet of baryons and mesons in the exit channels leading to a multitude of different reactions relative to only four $SU(2)$-cases. The latter are, however, more trustable, because pion masses are numerically small. We therefore will start with the pion-nucleon amplitudes and their modifications when going to SU(3).

**Pion production in $SU(3)$**

Table 1: Kroll-Ruderman amplitudes in units $10^{-3} m_{\pi^+}^{-1}$ for the cases:
(i) the standard low-energy theorem in $SU(2)$.
(ii) $SU(2)$ soliton model, ref.[2].
(iii) $SU(3)$ soliton model according to eqs.(10, 11, 12).
(iv) reanalysed experimental data, M: ref.[4], S: ref.[5].

| | standard LET | SU(2) soliton model | SU(3) soliton model | experiment |
|---|---|---|---|---|
| $A^{n(\gamma,\pi^-)p}$ | $-31.8$ | $-31.8$ | $-31.3$ | $-31.4 \pm 1.3^M$<br>$-32.2 \pm 1.2^S$ |
| $A^{p(\gamma,\pi^+)n}$ | $-27.4$ | $-27.4$ | $-27.9$ | $-27.9 \pm 0.5^M$<br>$-28.8 \pm 0.7^S$ |
| $A^{p(\gamma,\pi^0)p}$ | $-2.5$ | $-1.6$ | $-.8$ | $-2.0 \pm 0.2^M$<br>$-1.5 \pm 0.3^S$ |
| $A^{n(\gamma,\pi^0)n}$ | 0.4 | 1.6 | .6 | |

Table 1. shows the numerical effects of symmetrically adding the strange flavor degree of freedom to the rotating soliton. In this case the strangeness content of non-strange baryons is, of course, maximal ($\frac{7}{30}$). Because the leading term for charged pion amplitudes is independent from the number of flavors involved, deviations only arise from the subleading terms and are thus most easily noticeable in the amplitudes for neutral pion production. Of course, the assumption of $SU(3)$-symmetry is not given in the baryonic wave functions, see e.g. ref.[3], and here it only serves as a demonstration for the fact, that $SU(3)$ leads to a different and noticeable change coming from the anomalous terms of the action. Incidentally, the novel term $L_{\gamma\phi}^{(ano,2)}$, eq.(12), only amounts to roughly 10% of $L_{\gamma\phi}^{(ano,1)}$, eq.(11) and is unimportant in this respect. The latter, on the other hand, continuously returns to its $SU(2)$ value with decreasing strangeness content of the nucleons. I have already sketched this limit in the remark following eq.(11).

**Strangeness production**

Turning to the photoproduction of strangeness on the nucleon we are leaving safe ground, because several reasons lead to numerically uncontrolable statements. Nevertheless, the amplitudes derived allow for some qualitative insight.

Table 2: The Born coupling constants for reactions with strangeness production as given by three flavor soliton models from eq.(10). The first column shows the case for $SU(3)$-symmetry of the baryon wave functions, the second column uses realistic symmetry breaking for the baryons, and the last column shows the effect of a formfactor as explained in the text. Quantities with a '*' are normalized to their empirical value.

| | SU(3) symmetry | broken SU(3) | FF effects |
|---|---|---|---|
| $g_{\pi NN}/\sqrt{4\pi}$ | $3.78^*$ | $3.78^*$ | – |
| $g_{K\Sigma N}/\sqrt{4\pi}$ | $-1.08$ | $-.89$ | $\sim 0$ |
| $g_{K\Lambda N}/\sqrt{4\pi}$ | 3.74 | 2.69 | $\sim 0$ |

Table 2. displays the Born couplings, as they can be deduced from soliton models taking Euler-angle matrixelements over eq.(10) for different baryons. For $SU(3)$-symmetry, first column, they coincide with conventional expresssions, ref.[6]. These couplings have been considered as being far too large, to accomodate for the strangeness production data [7], and other strategies have then been attempted there.

The soliton model allows for more sophisticated approaches:
(i) one may relax the assumption of $SU(3)$-symmetry on the baryon wave functions and use those diagonalizing the $SU(3)$-symmetry breaking. Using e.g. the rigid rotator approximation[3] one may see from the second column of table 2., that this leads to a reduction of the born couplings.
(ii) the soliton model offers, in a natural way, formfactors to these couplings: the axial current in eq.(7) only leads to the axial charge in eq.(10) when integrated as in eq.(9) with a constant photon field. Inclusion of non-zero wavenumbers $k$ of the photon introduces an additional $j_0(kr)$ into the integral, leading to $g_A(k)$. Of course, this k-dependence is beyond the order of the amplitudes restricted to here, i.e. $\mathcal{O}(m_K)$, but numerically most important: with this inclusion the strange Born couplings are all totally suppressed at kaon threshold, column 3. in table 2. Similar things will happen for the other two contibutions, eqs.(11,12), not considered here.

The conclusion from the soliton model is thus, that the expansion of strangeness production amplitudes is not possible in the way attempted here and much more effort must be invested. I will briefly sketch this effort: the assumption that the kaon fluctuation used here is a constant chiral rotation at $K\Lambda$ or $K\Sigma$ threshold is not justified. In meson-baryon scattering these channels plus additional ones, predominantly $\pi N$, $\eta N$ all couple with each other. In the, to my knowledge, only existing scattering calculation[9] in soliton models with broken $SU(3)$ I had suggested that $K\Lambda$, $K\Sigma$ and $\eta N$ form a bound state below $\eta N$ threshold. (Closing the $\pi N$-channel artificially I can actually calculate such a bound state at roughly 450MeV above the nucleon mass). If true, such a complex bound state way below the $K\Lambda$ or $K\Sigma$ threshold must strongly distort the kaonic fluctuations to something far from the constant chiral rotation I attempted to use here at threshold. Therefore, the photoproduction amplitudes for strangeness production can only be obtained after solving the meson-baryon scattering problem then inserting the obtained meson waves into the one photon production vertex as has been done for the case of $SU(2)$, refs.[8]

Incidentally, this bound state explains the known strong $\eta$ decay branching of the S11(1535) resonance[9] (and similar ones with higher strangeness, e.g. $K\Xi$ in $\bar{K}N$- scattering). The $\pi$ baryon, $\eta$ baryon channels have practically no direct coupling in soliton models. A similar statement has been made recently[10] in connection with chiral perturbation theory.

### Eta production

Almost identical reasons as put forward in the case of strangeness production apply to $\eta$ production: a constant chiral $\eta$-rotation cannot be sufficient in the presence of strongly coupled bound meson-baryon channels ($K\Lambda$, $K\Sigma$ and $\eta N$) already present below $\eta N$ threshold.

### Conclusion

In solitonic approaches the low-energy pion S-wave is given explicitly as a constant chiral rotation orthogonal on the soliton translation. The well-known low-energy theorem for pion photoproduction, following from this fact acquires a slightly different form in the subleading terms (production of neutral pions) when strangeness degrees of freedom are included for mesons and the soliton.

Low-energy theorems for pion production on strange baryons are then also derivable and numerically trustable.

Production amplitudes for the heavier mesons obtained by the same method as for pions are not trustable, because the S-wave resonances strongly violate the assumption of the heavy meson wave being a chiral rotation at threshold. I have argued, that this violation and the $\eta$-decay of some S-wave resonances are

closely connected due to the existence of bound states of strangeness +1 kaons with strange baryons.

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# References

[1] P. de Baenst *Nucl. Phys.* **B24** (1970) 633

[2] B. Schwesinger and H. Walliser *Nucl. Phys.* **A574** (1994) 836

[3] H. Weigel *Intl. Journ. Mod. Phys.* **A11** (1996) 2419, and references therein.

[4] A. M. Bernstein and B. R. Holstein *Comm. Nucl. Part. Phys.* **20** (1991) 197

[5] J. Bergstrom *Phys. Rev.* **C44** (1991) 1768

[6] T. Mart, C. Bennhold and L. Alfieri **nucl-th/9606024**

[7] T. Mart, C. Bennhold and C. E. Hyde-Wright *Phys. Rev.* **C51** (1995) R1074

[8] G. Eckart and B. Schwesinger *Nucl. Phys.* **A458** (1986) 620
B. Schwesinger, H. Weigel, G. Holzwarth and A. Hayashi *Phys. Rep.* **173** (1989) 173

[9] B. Schwesinger *Nucl. Phys.* **A537** (1992) 253

[10] N. Kaiser, P. B. Siegel and W. Weise *Phys. Lett.* **B362** (1995) 23